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Gfg2 Summer School

Signals, Systems and Signal Structure

Lecturer: Andy Sowter
Department of Civil Engineering

Topics Covered

Analogue Signals

Digital Signals

Digital Coding

Structure of Transmitted Signal

Extraction of Information at the Receiver

Lecture 1

- Analogue and Digital Signals – how do we characterise them?
- Different ways of using these properties for positioning (special emphasis on Satellite Positioning)

Analogue Signals

These are <u>continuous</u> functions of time.

Example : Cosine Signal

$s(t) = A \cos(\omega t + \phi)$

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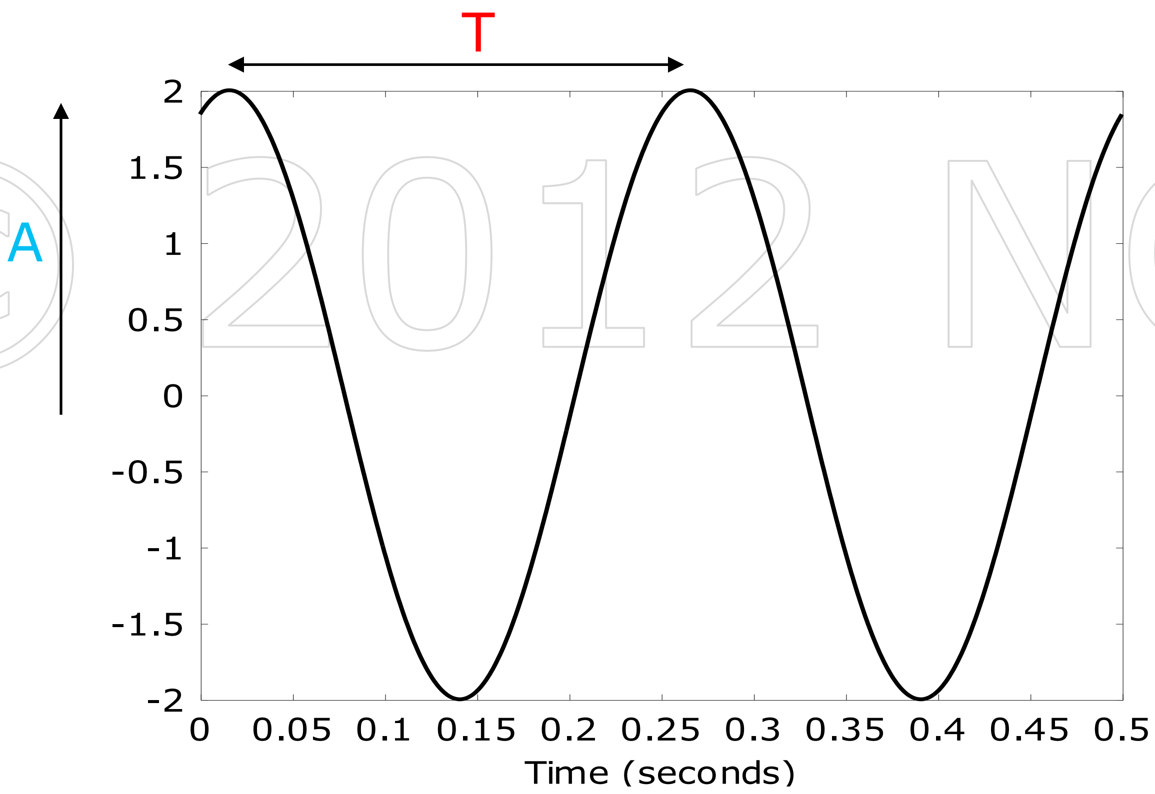
Analogue Signals

$$s(t) = A\cos(\omega t + \phi)$$



Amplitude, Period and Frequency

A is the amplitude of the signal

T is the period of the signal

$$s(t+T) = s(t)$$

The angular frequency is defined as

$$\omega = \frac{2\pi}{T} \quad rad\ s^{-1}$$

The cyclical frequency is defined by

$$f = \frac{\omega}{2\pi} = \frac{1}{T} \quad Hz$$

Wavelength and its Relation with Frequency

c = speed of light = 3×10^8 ms^{-1}

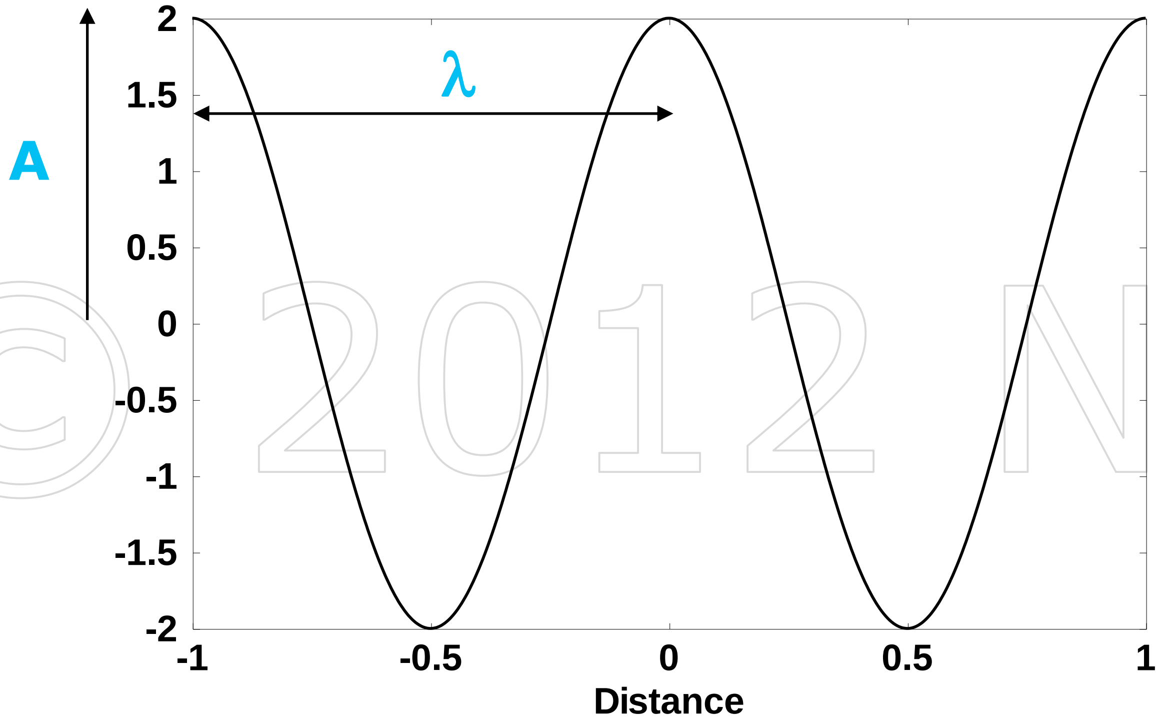
Now in one period of the signal (T) the wave will travel a distance

$$\lambda = cT$$

where λ = wavelength.

and $$T = \frac{1}{f}$$

Wavelength and its Relation with Frequency

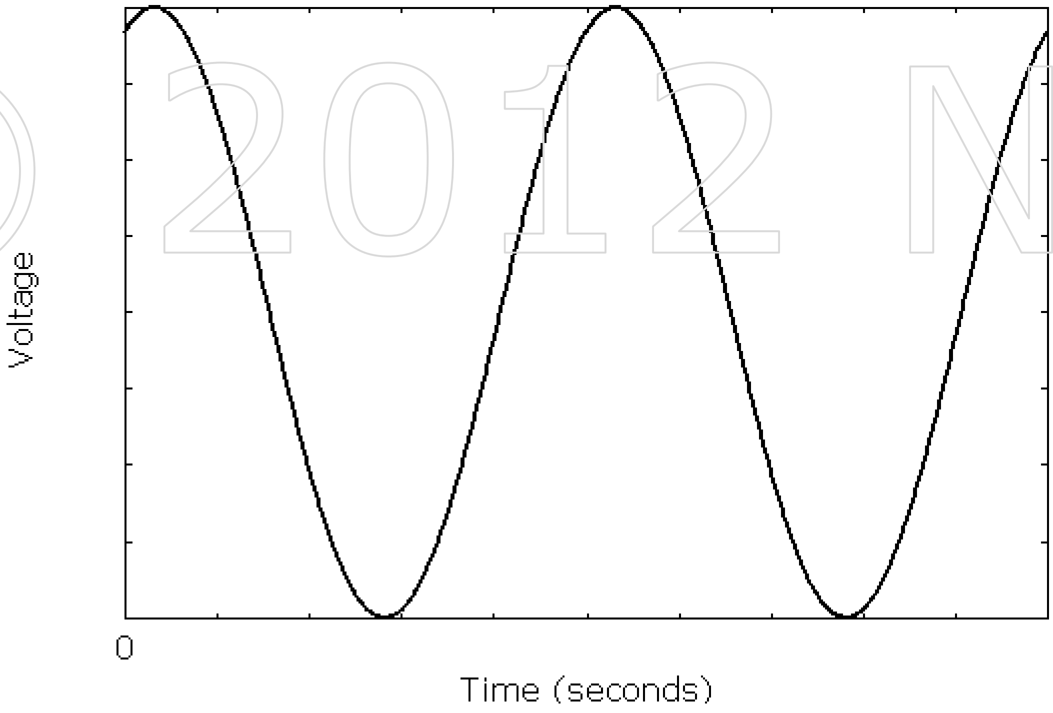


$$c = f\lambda$$

i.e. **speed = frequency x wavelength.**

Spectral Analysis

$$s(t)=A.\cos(2\pi ft + \phi)$$

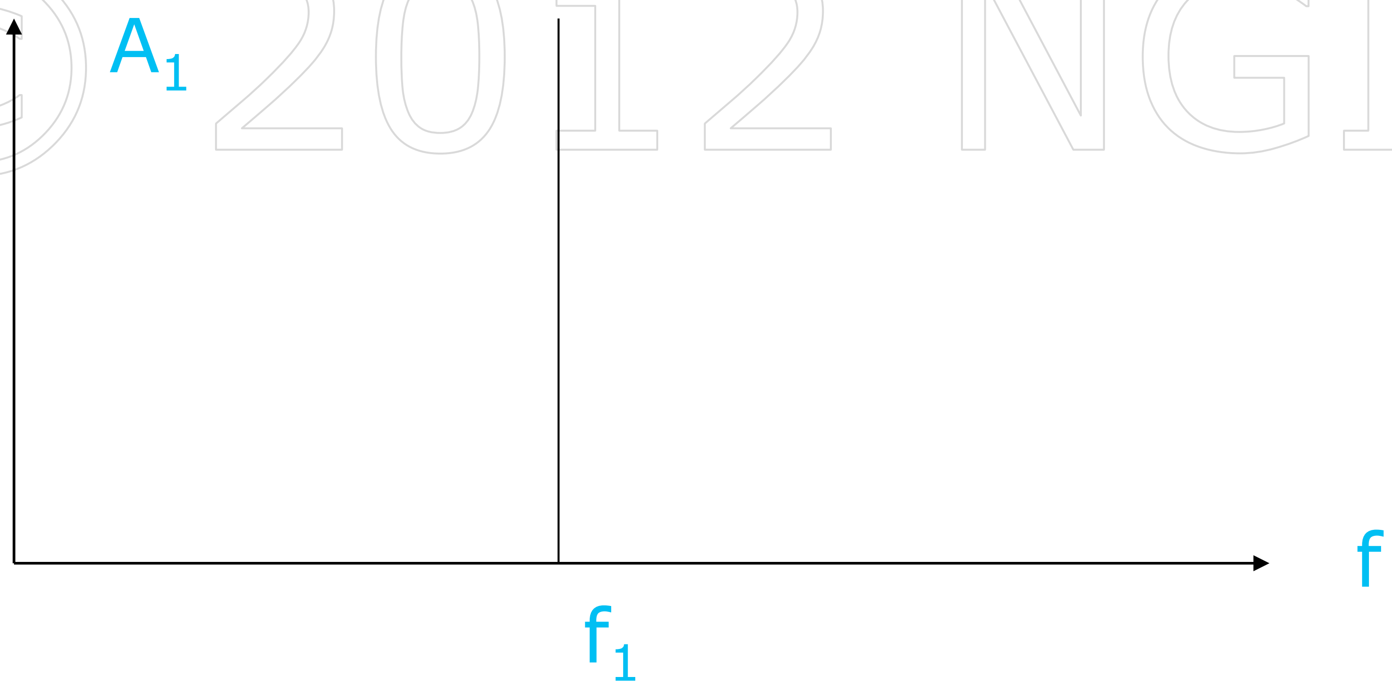


Spectral Analysis

$$s(t)=A_1.\cos(2\pi f_1 t + \phi_1)$$

Signal has amplitude A_1 and frequency f_1

Represent as an "Amplitude Spectrum"

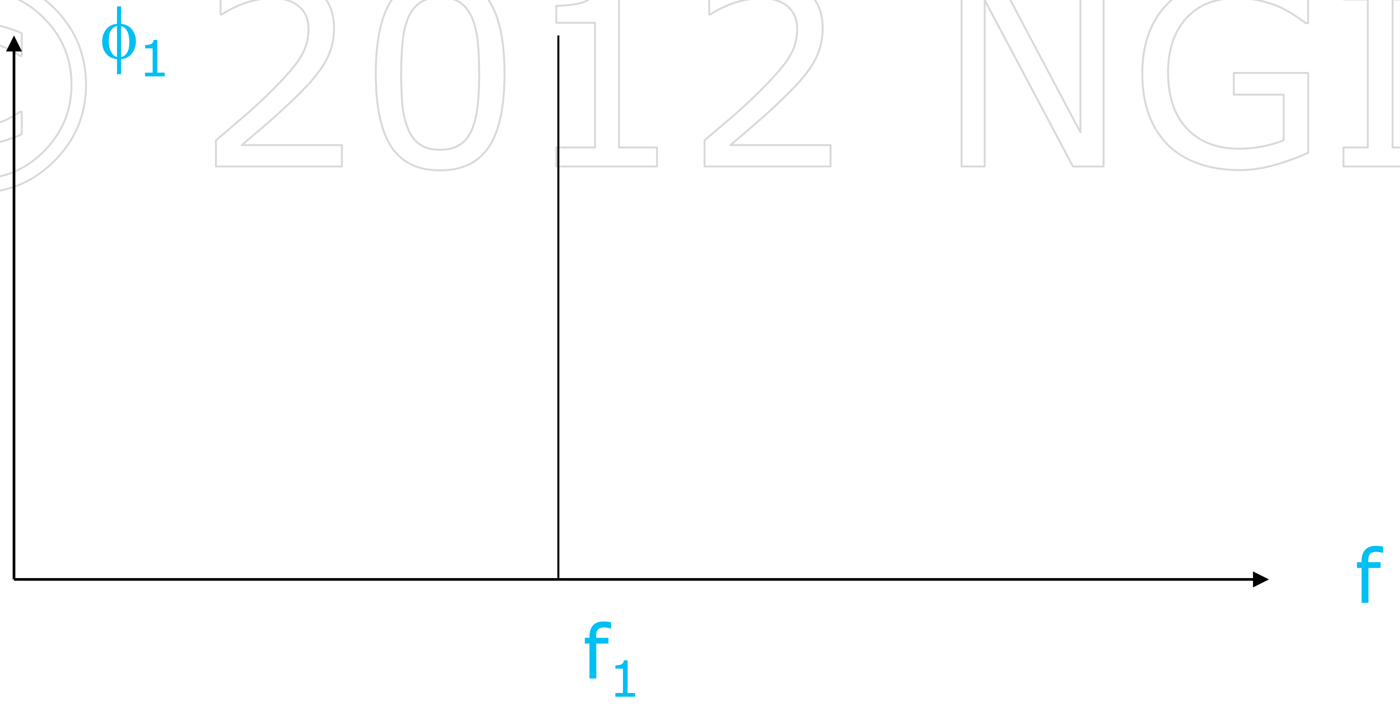


Spectral Analysis

$$s(t)=A_1.\cos(2\pi f_1 t + \phi_1)$$

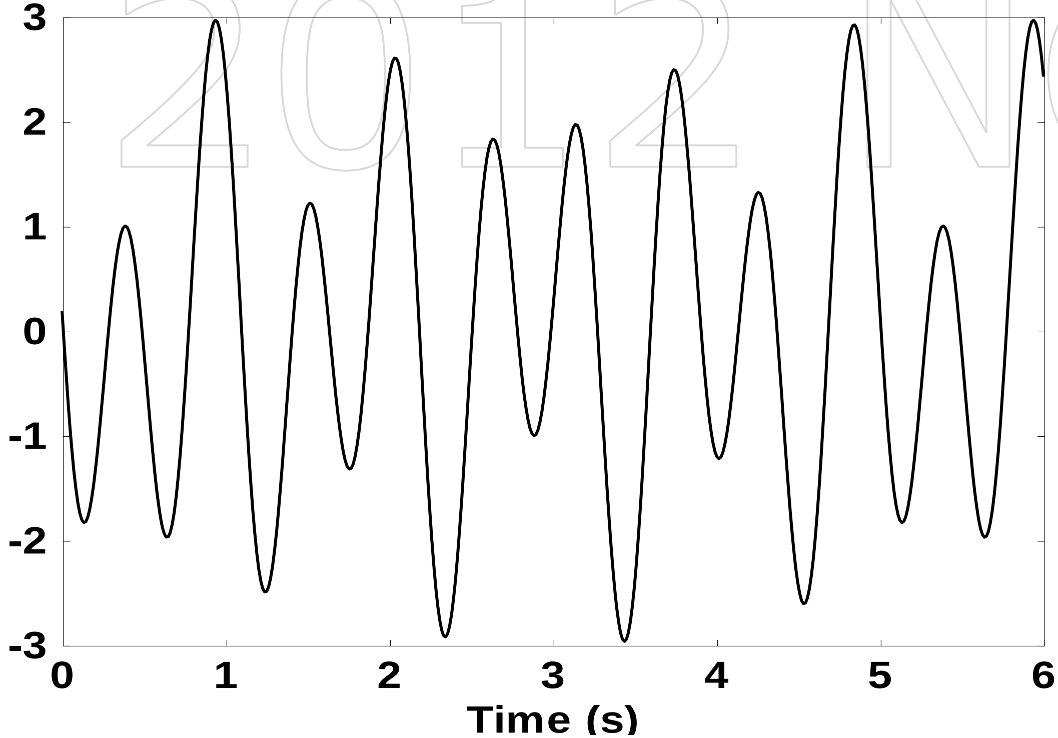
Signal has phase ϕ_1 and frequency f_1

Represent as a "Phase Spectrum"



Can be extended to two or more frequencies, e.g.

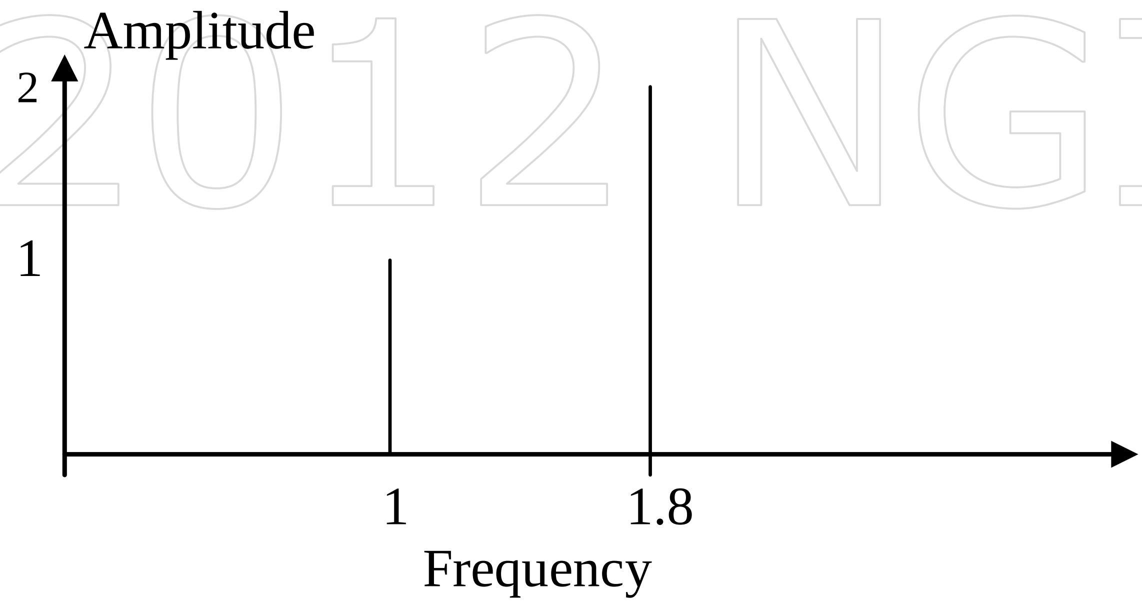
$A_1 = 1$, $A_2 = 2$, $f_1 = 1$ Hz, $f_2 = 1.8$ Hz, $\phi_1 = 0.628$ radians, $\phi_2 = 1.885$ radians.



Spectral Analysis

$A_1 = 1$, $A_2 = 2$, $f_1 = 1$ Hz, $f_2 = 1.8$ Hz, $\phi_1 = 0.628$ radians, $\phi_2 = 1.885$ radians.

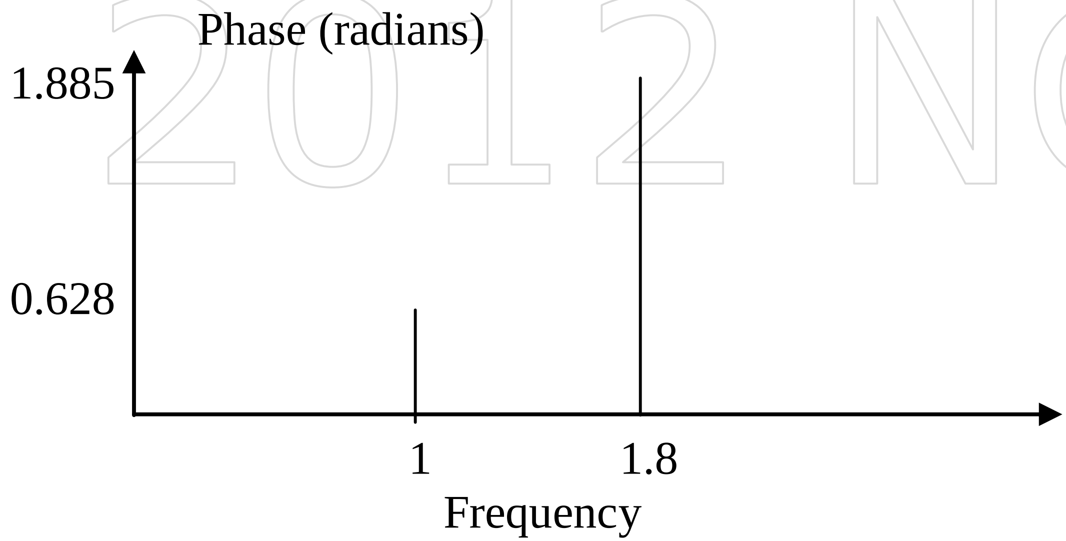
Amplitude Spectrum



Spectral Analysis

$A_1 = 1$, $A_2 = 2$, $f_1 = 1$ Hz, $f_2 = 1.8$ Hz, $\phi_1 = 0.628$ radians, $\phi_2 = 1.885$ radians.

Phase Spectrum



Spectral Analysis

Measure amplitude and phase spectra with a SPECTRUM ANALYSER

In many applications, AMPLITUDE SPECTRUM, is usually more important than the phase spectrum

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Bandwidth of a Signal

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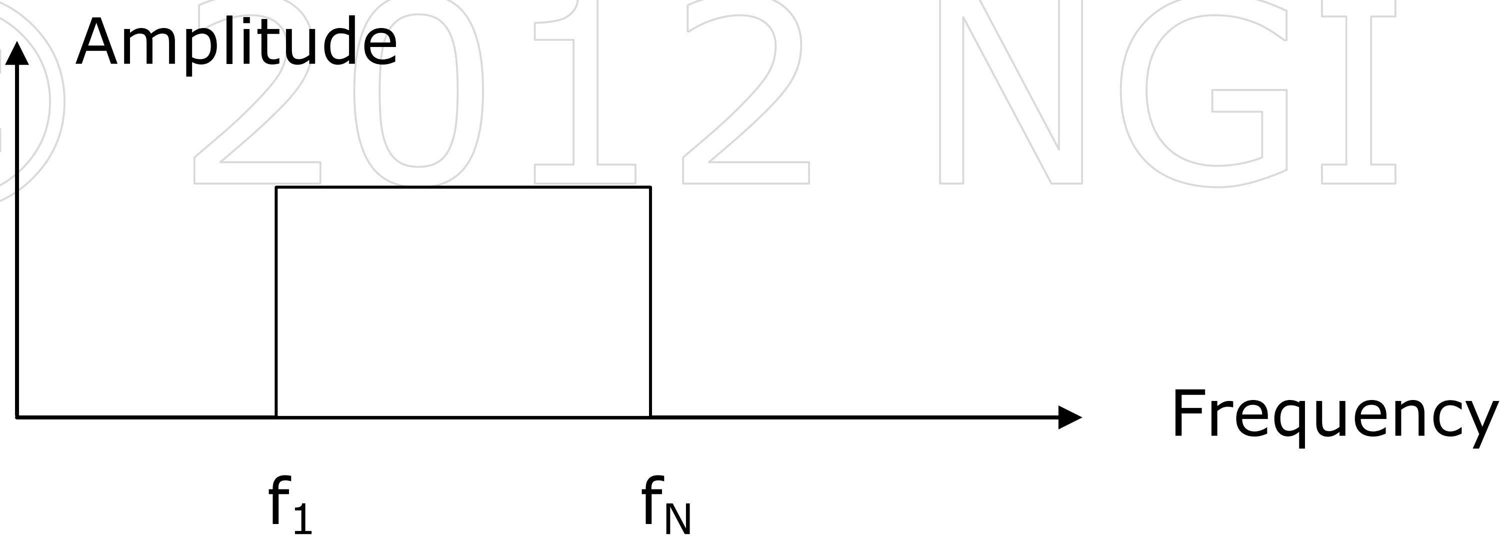
In this case, the bandwidth of the signal is $f_N - f_1$ Hz.



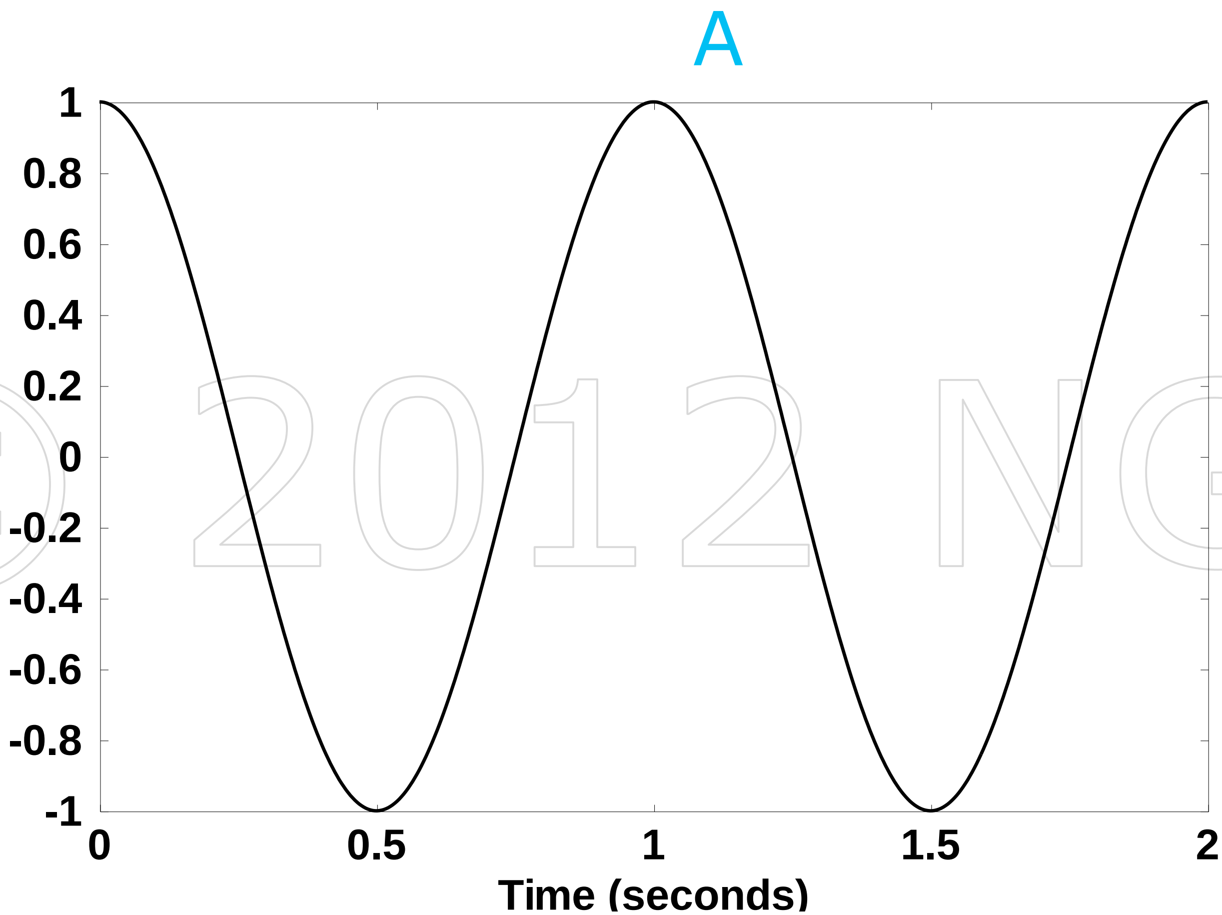
Phase

$$s(t) = A \cos(\omega t + \phi)$$

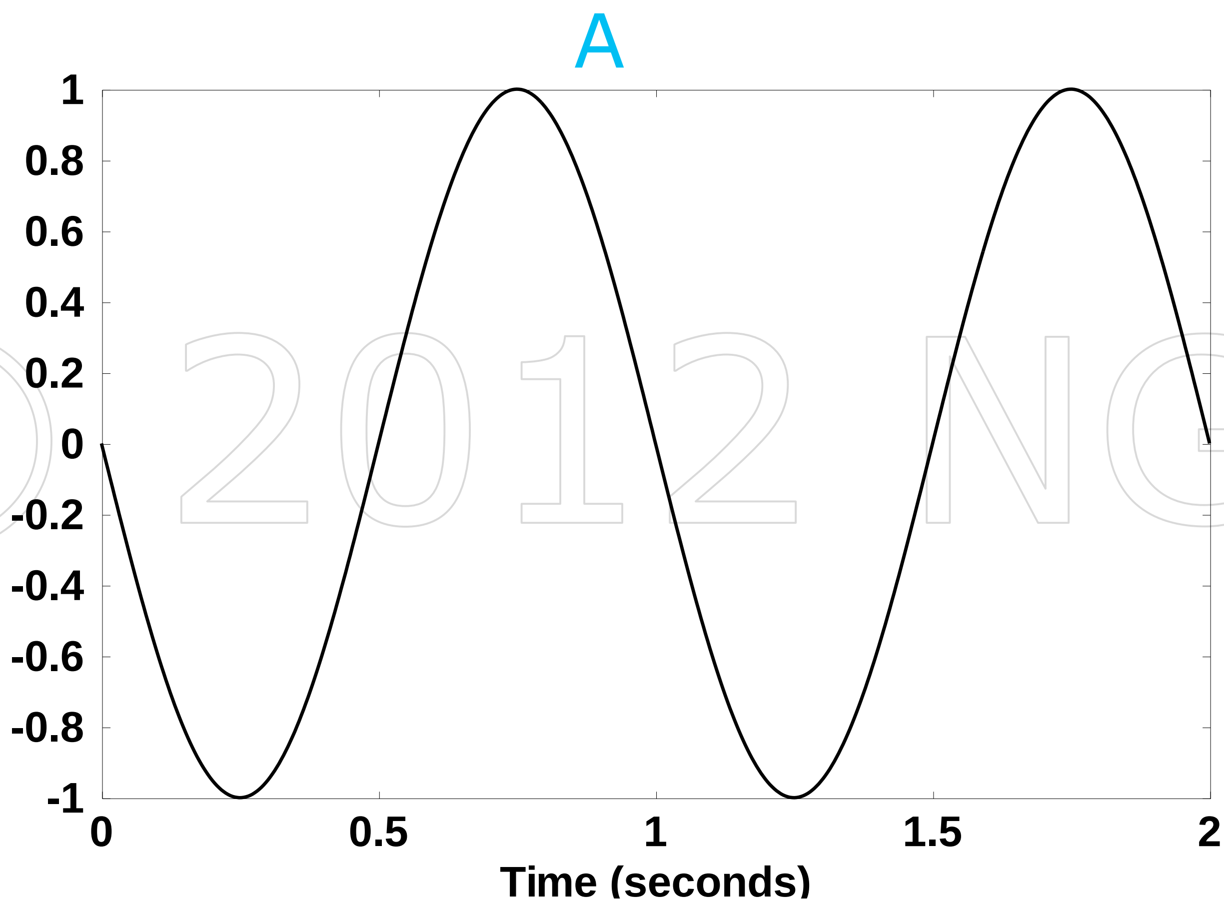
What happens to the signal as we change ϕ ?

Look at time reference A in the following diagrams

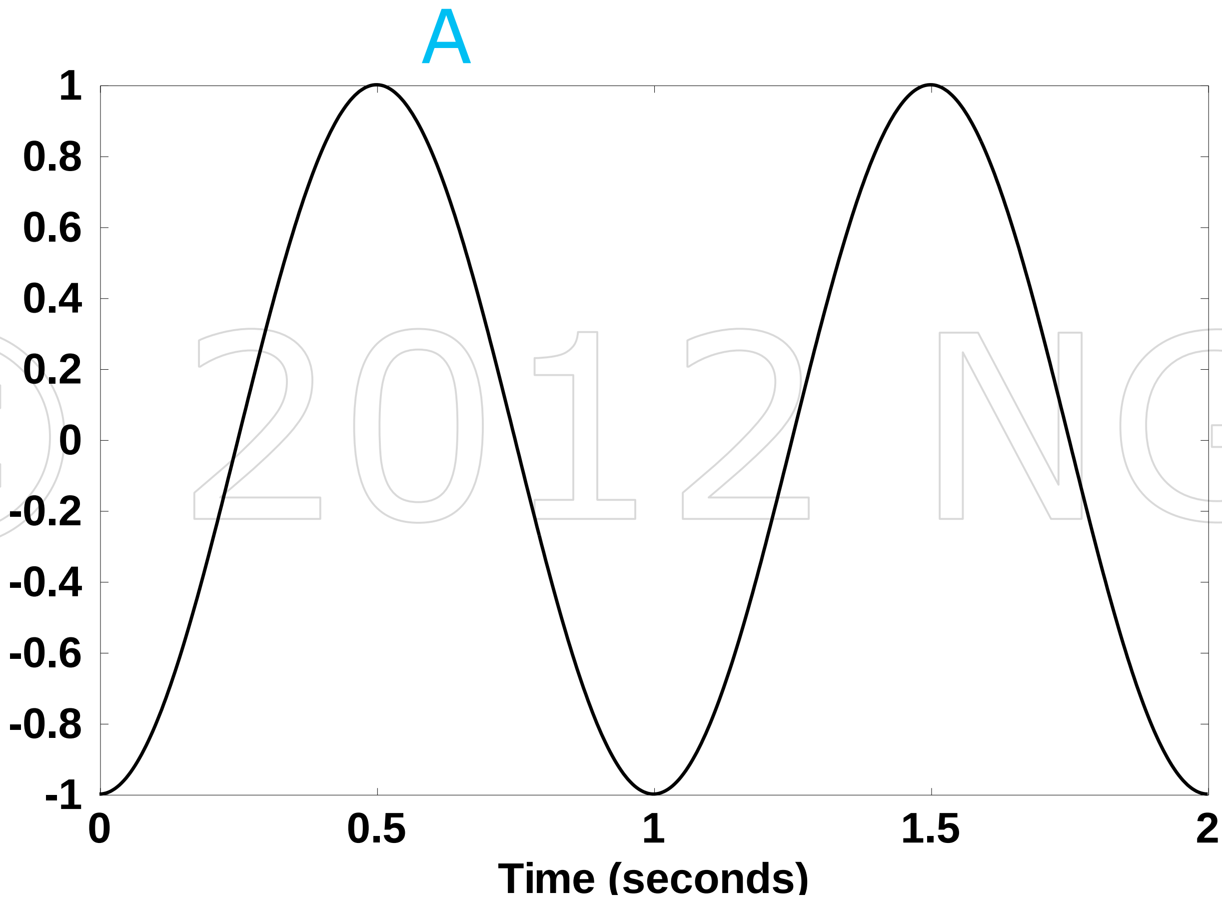
$\phi = 0$



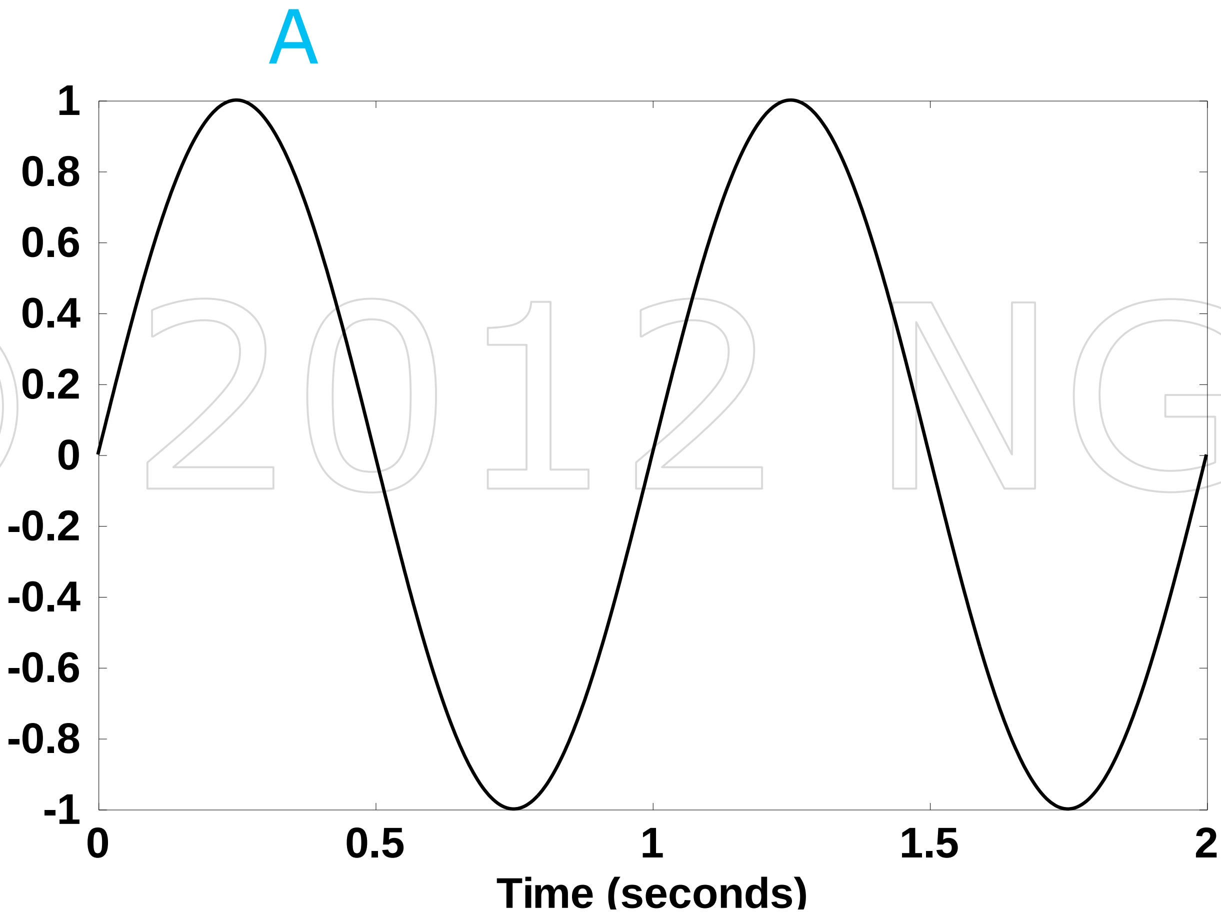
$\phi = \pi/2$



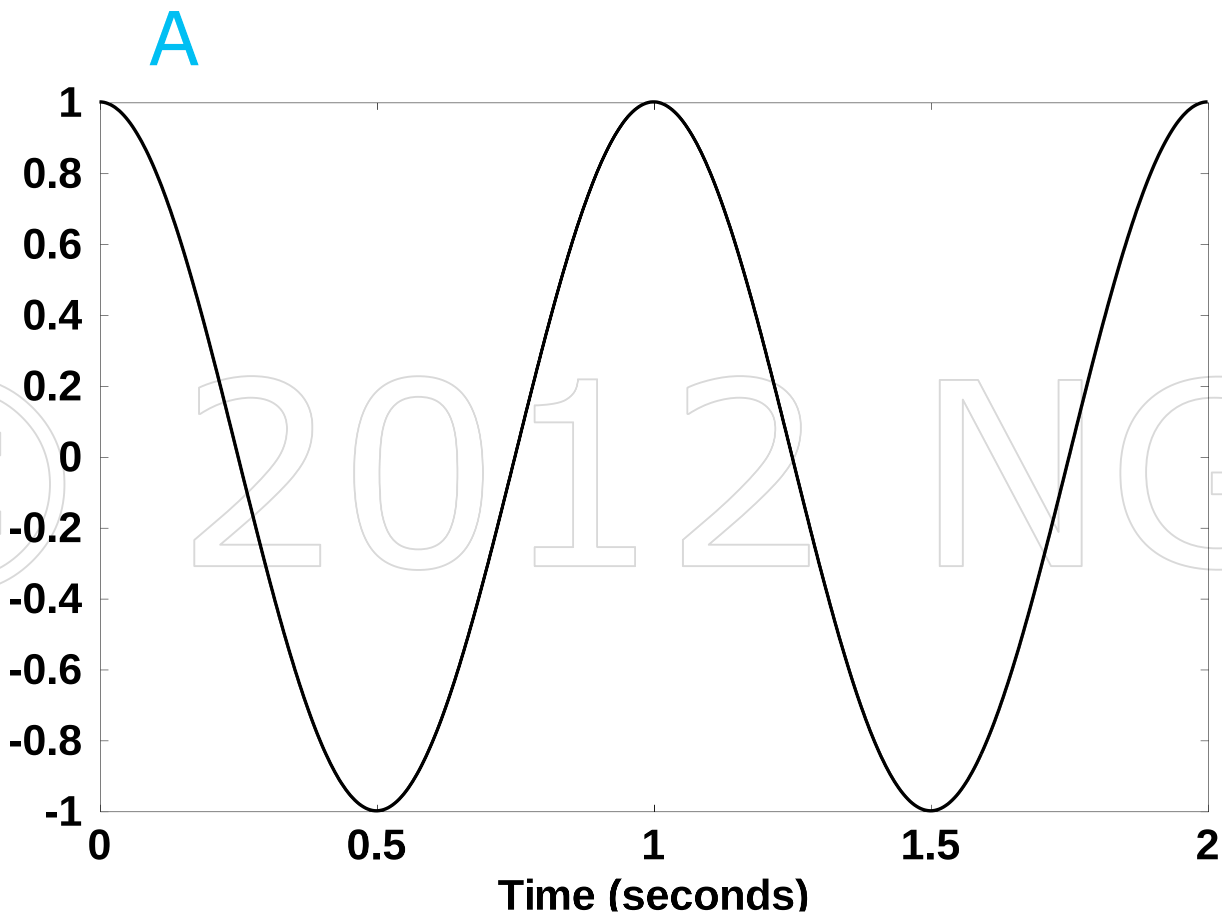
$\phi = \pi$



$\phi = 3\pi/2$



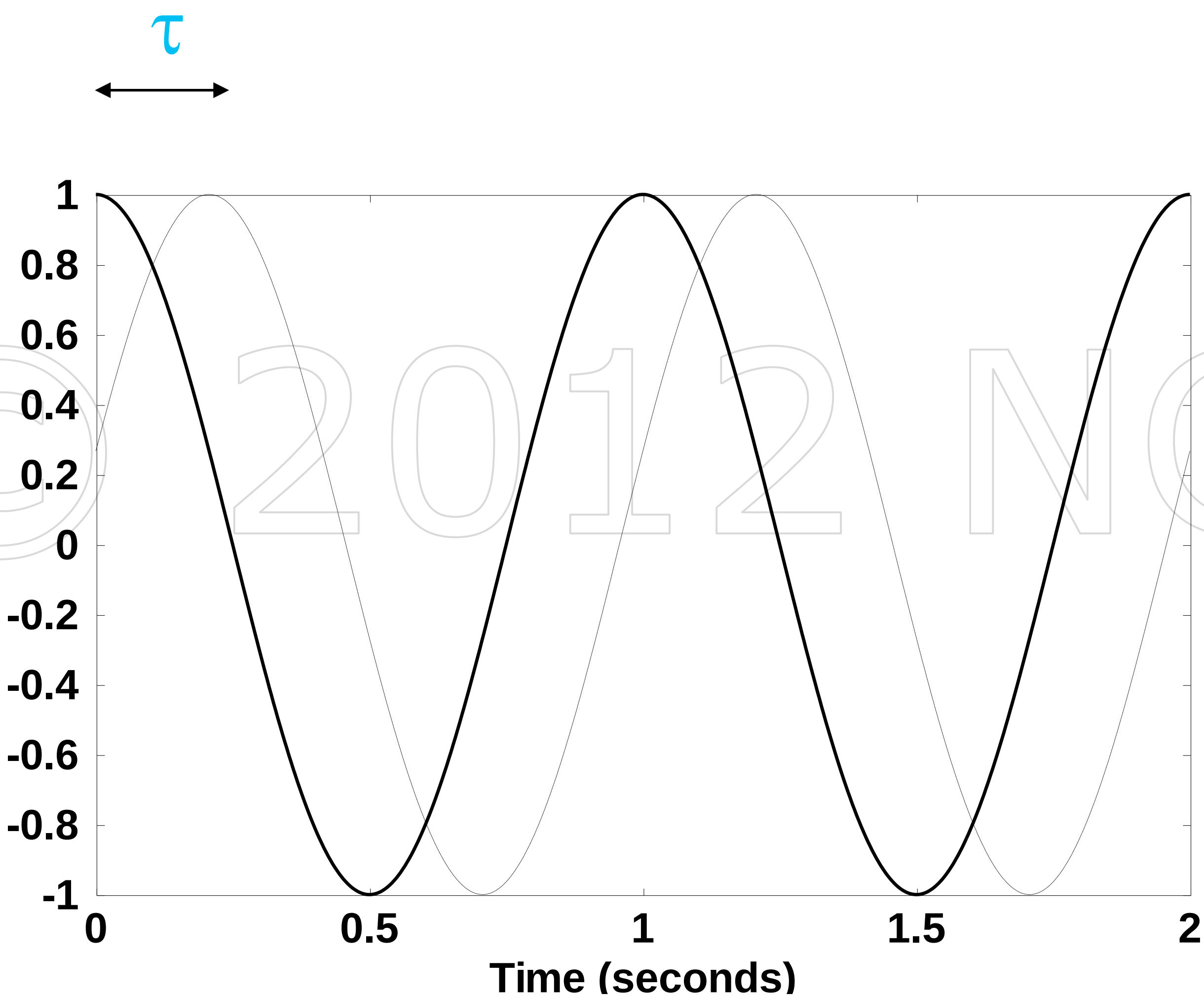
$\phi = 2\pi$



Phase

- Change in Phase = Shift in Time
- Change of Phase $\pi/2$ = shift in time T/4 (T=period)
- Shift of Phase of 2π = shift in time of T. Signal stays the same
- So cannot distinguish between a phase of 0 and a phase of 2π
- Adding a phase that is a multiple of 2π leaves the signal the same

General Time Delay



General Time Delay

How is τ related to ϕ ?

Full curve: $s_1(t)=\cos(\omega t)$

Dashed Curve:

$s_2(t)=\cos(\omega(t-\tau))$

General Time Delay

$$s_2(t) = \cos(\omega(t - \tau))$$

$$= \cos(\omega t - \omega\tau)$$

$$= \cos(\omega t + \phi)$$

where $\phi = -\omega\tau$

General Time Delay

Thus, a time delay τ is equivalent to a phase change - $\omega\tau$.

Example :

f= 2 Hz ; τ = 0.125 s

$\phi = -\omega\tau = -2\pi f\tau = -4\pi/8 = -\pi/2$ rads

General Relation between Wavelength and Distance

$$\frac{\phi}{2\pi} = \frac{x}{\lambda}$$

$$x = \frac{\phi\lambda}{2\pi}$$

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Ambiguities in Determining Position

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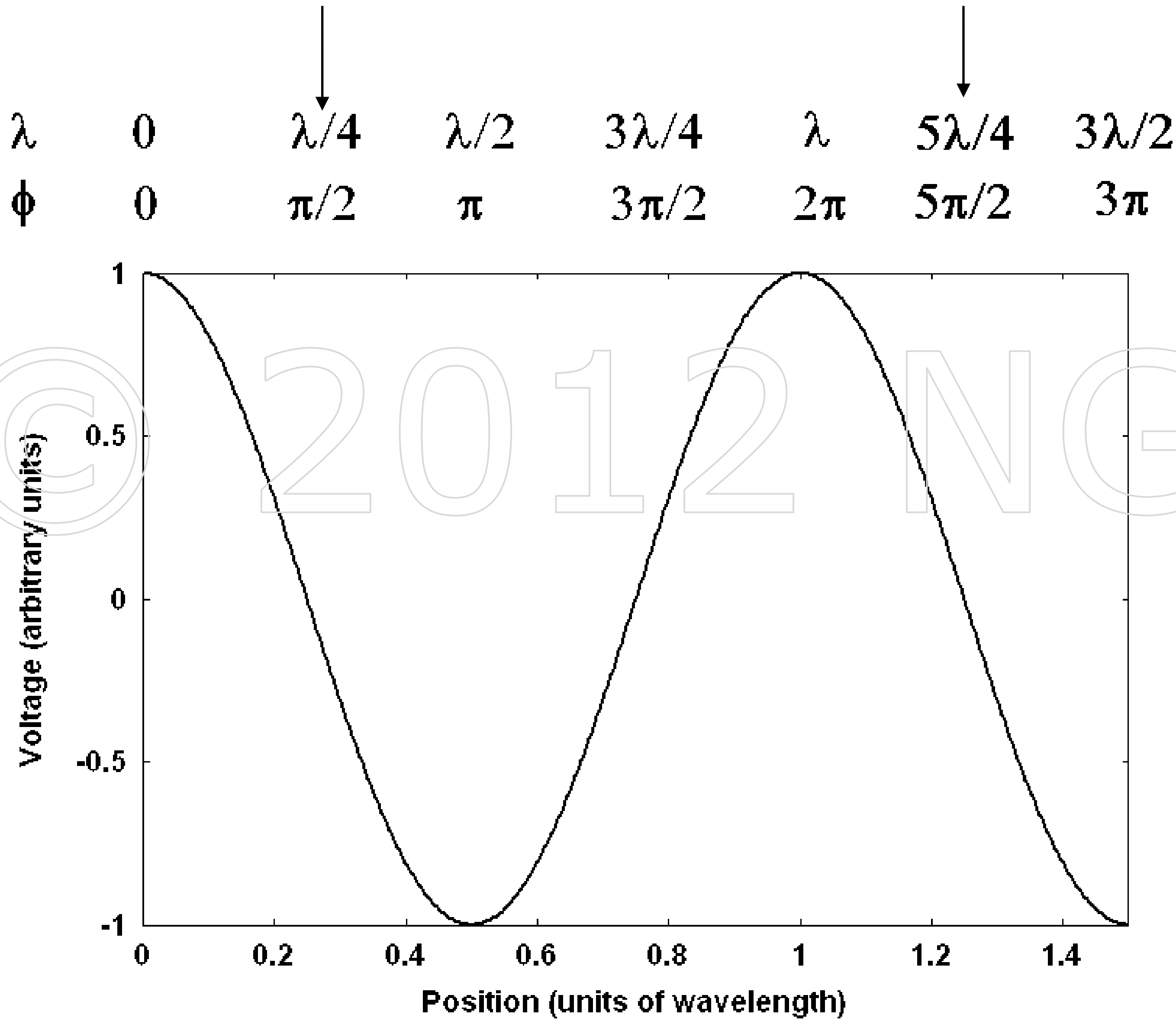
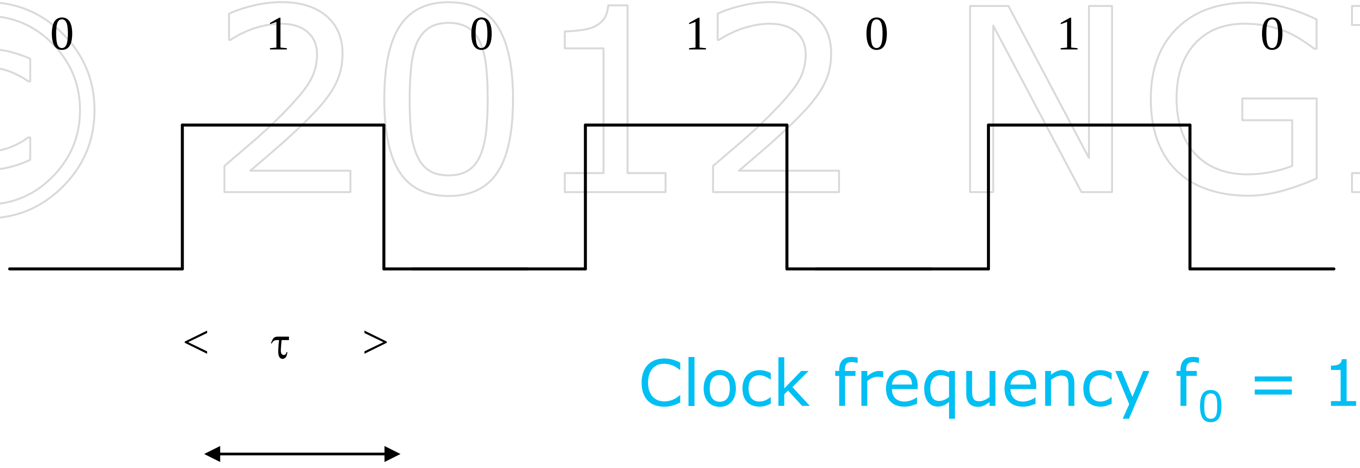


Digital Signals

The message and ranging codes used in GPS and other positioning systems are digitally coded signals



Clock frequency $f_0 = 1/\tau$ Hz

Each '1' or '0' is called a "bit" or "chip"

Equivalent distance of one chip $\Delta R = c\tau$

GPS Digital Signal Parameters

Basic Clock frequency f_0 = 10.23 MHz

P code

Clock frequency f_0

$\Delta R = 3 \times 10^8/(10.23 \times 10^6) \approx 29$ m

Period = 1 week

C/A code

Clock frequency $f_0/10$

$\Delta R \approx 290$ m

Period = 1 ms

Pseudorandom Noise (PRN,PN)

- P- and C/A- codes look like noise to the outside world.
- Codes stored in receiver
- Refer to each '1' or '0' in these codes as a "chip"

Message

Message:

Clock frequency = $f_0/204600 = 50$ Hz

- Parameters to correct satellite's clock errors
- orbital data
- ionospheric model (single frequency use)
- refer to each '1' or '0' in the message as a "bit"

Lecture 1

Analogue Signals: Frequency, Wavelength, Amplitude

Spectral Analysis

Digital Signals

Components of GPS Signals (Compass, Galileo)

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Lecture 2

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Digital coding and its use in satellite positioning

Digital Correlation

Advantages of correlation in satellite positioning

Transmission of digital signals – modulation of high frequency carrier

Extraction of positioning and other information at the receiver

Digital Coding and Modulation

Used in Global Navigation Satellite Systems (GNSS) to determine position

Also used in radar to obtain fine resolution images

Digital Coding

Signal is divided into segments called "chips"

Each chip takes on the value $\cos(\phi_n)$ where $\phi_n = 0$ or π

When $\phi_n = 0$, $\cos(\phi_n) = 1$

When $\phi_n = \pi$, $\cos(\phi_n) = -1$

Digital Coding

In one convention $\phi_n = 0$ is denoted as '0'

and $\phi_n = \pi$ is denoted as '1'

In another convention, $\phi_n = 0$ is denoted as '+'

and $\phi_n = \pi$ is denoted as '-'

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Digital Coding

0	0	1	0	1

For example: 00101
Can also be written as

+	+	-	+	-

Digital Correlation

Digital Correlation is a technique that is central in GNSS to both identify transmissions from each satellite and also to determine range.

Correlation measures how similar two coded pulses are

Digital Correlation Example

Suppose that we have the following two coded pulses

+	+	-	+	-
-	+	+	-	-

Define **Correlation Function**:

(1) If two chips are identical this contributes +1 to the correlation function

(2) If two chips are different, this contributes -1 to the correlation function

(3) Add the +1's and -1's to obtain the correlation function

Digital Correlation Example

Suppose that we have the following two coded pulses

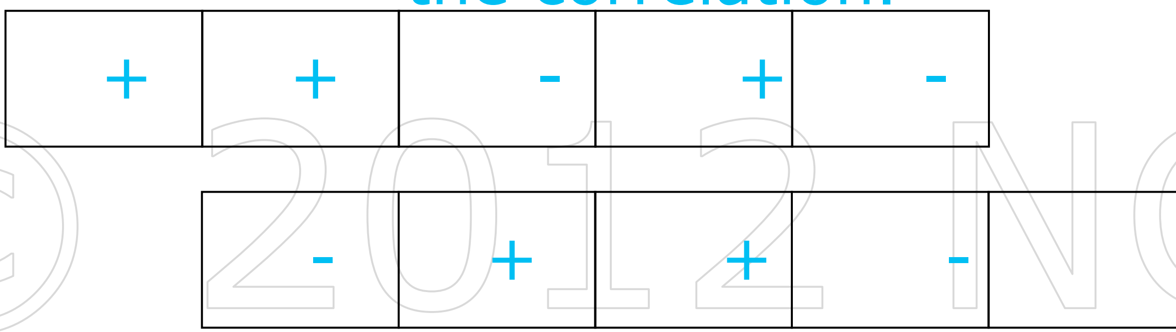
+	+	-	+	-
-	+	+	-	-

R(0) = -1 +1 -1 -1 +1 = -1

Digital Correlation Example

Shift lower pulse one chip to the right.

If a chip aligns with nothing then this contributes 0 to the correlation.

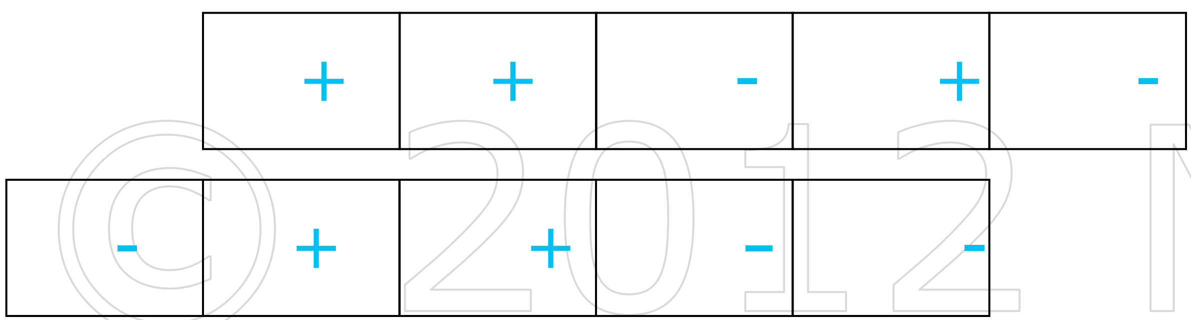


R(1) = -1 -1 +1 +1 = 0

This procedure is repeated for further shifts to the right to compute R(2), R(3), etc.

Digital Correlation Example

Can also shift lower pulse one chip to the left to calculate R(-1).

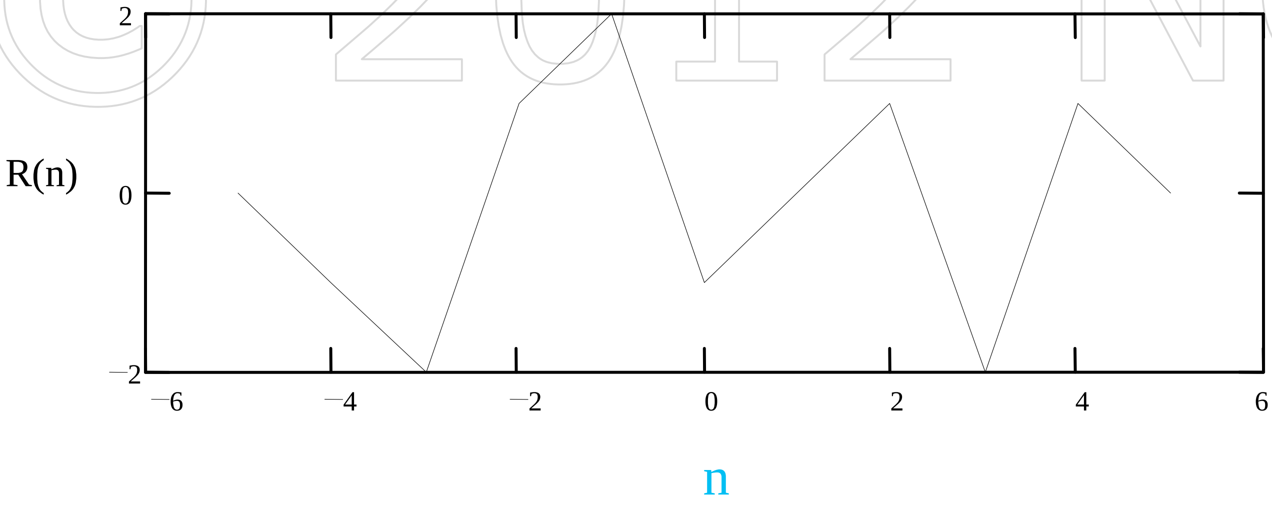


R(-1) = +1 +1 +1 -1 = 2

This procedure is repeated for further shifts to the left to compute R(-2), R(-3), etc.

Correlation Function

$\{R(n)\} = \{-1, -2, 1, 2, -1, 0, 1, -2, 1\}$



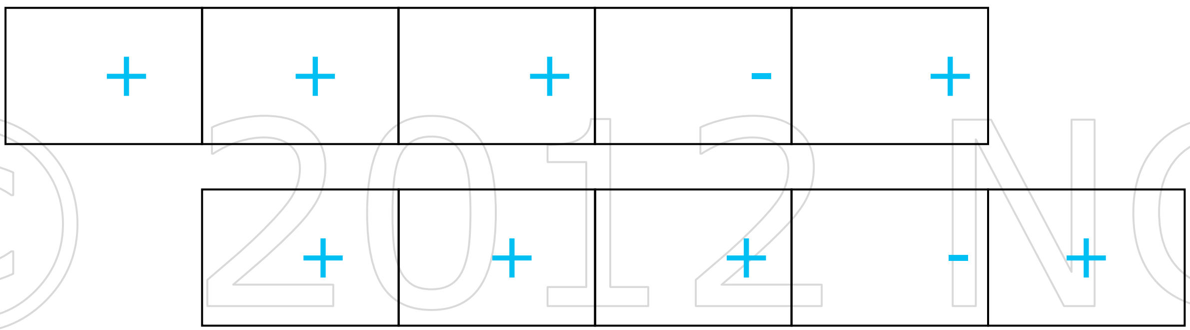
Digital Correlation Function

Now look what happens if we correlate a coded signal with itself

+	+	+	-	+
+	+	+	-	+

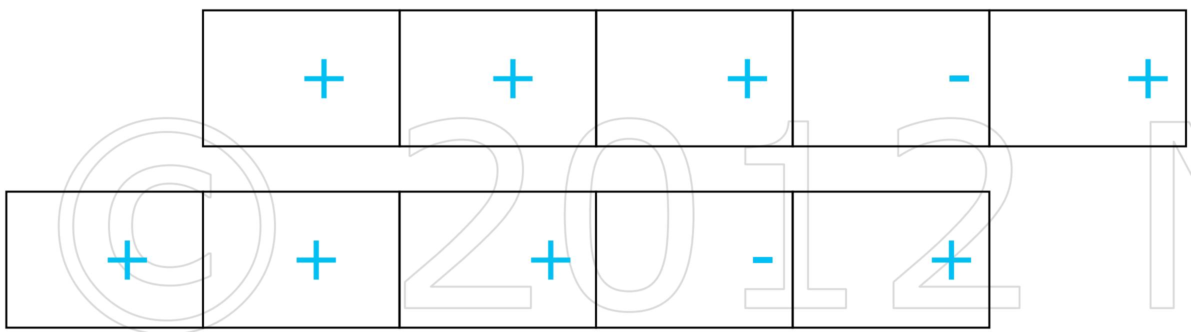
R(0) = +1 +1 +1 +1 +1 = 5

Digital Correlation Function



R(1) = +1 +1 -1 -1 = 0

Digital Correlation Function

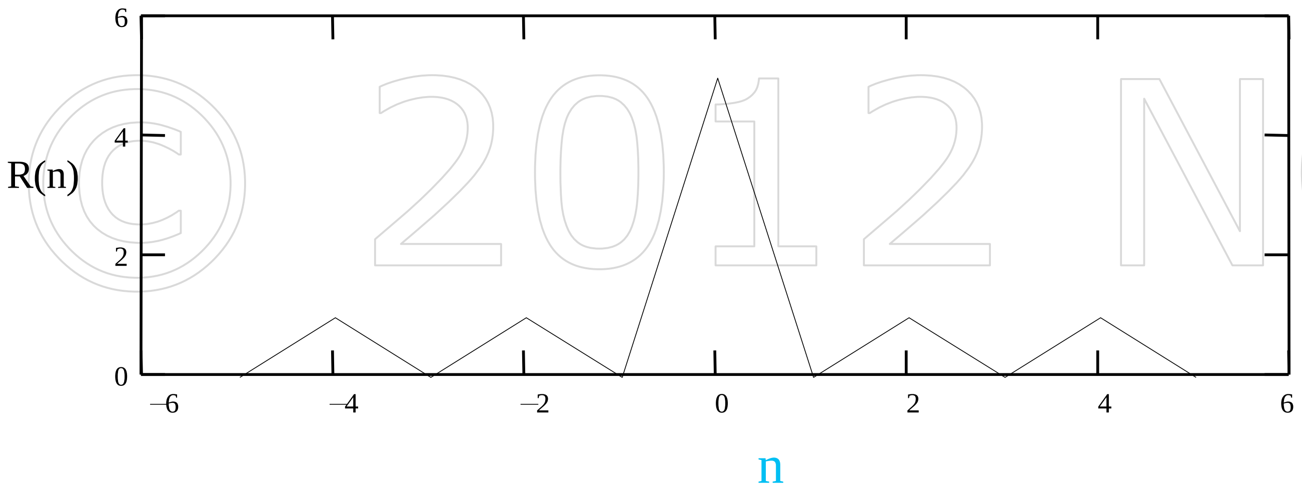


R(-1) = +1 +1 -1 -1 = 0

etc.

Correlation Function

$\{R(n)\} = \{1,0,1,0,5,0,1,0,1\}$



Autocorrelation and Cross Correlation Functions

If correlating two different signals, obtain what is known as the Cross-correlation function

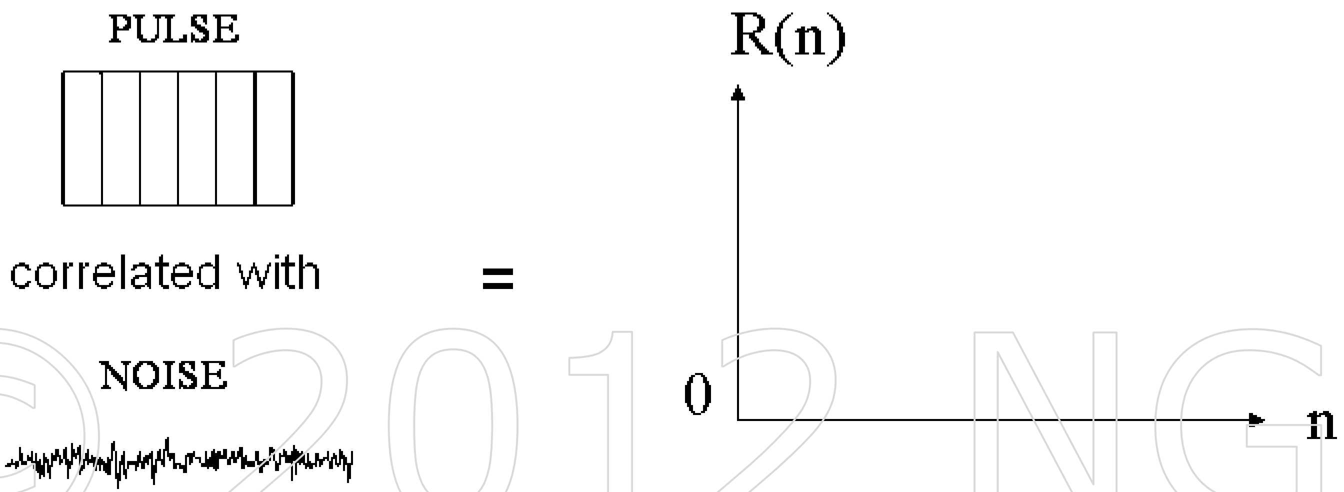
If correlating a signal with itself, obtain what is known as the autocorrelation function.

Autocorrelation function properties:

(1) R(n) = R(-n)

(2) R(0) = number of chips

Performance in Noise



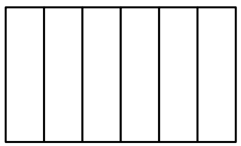
Important property of correlation:

If you correlate a pulse with noise, then the cross-correlation function is zero.

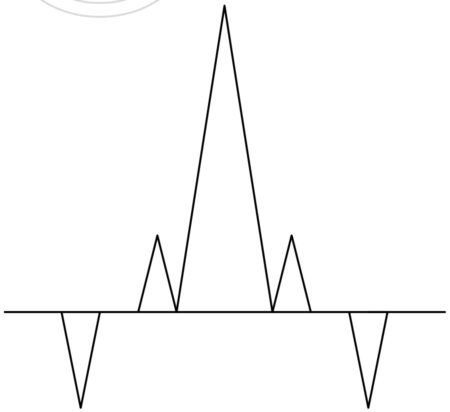
Performance in Noise

NOISY PULSE

CLEAN PULSE + NOISE



CORRELATED WITH CLEAN PULSE STORED IN RECEIVER



+ 0

= **AUTOCORRELATION FUNCTION OF CLEAN SIGNAL**

RANGE DETERMINATION

<u>Transmitted Signal (from Satellite)</u>

… + - + + - + - - - + + - +… →

<u>Signal Stored in Receiver (Template)</u>

… + - + + - + - - - + + - +… →

In this simple example, assume that code repeats every 13 chips

Range Determination

ON THE GROUND

<u>Signal Received From Satellite</u>

... + - + + - + - - - + + - +...

<u>Template in the Receiver</u>

- + + + - + + - + - - - +

By the time satellite signal has reached ground, receiver signal has advanced by a few chips

Range Determination

SHIFT RECEIVED SIGNAL TO THE RIGHT BY ONE CHIP

... + + - + + - + - - - + + - ...

- + + + - + + - + - - - +

Correlation = -3

Range Determination

SHIFT RECEIVED SIGNAL TO THE RIGHT BY TWO CHIPS

... + + + - + + - + - - - + + ...

- \+ + + - + + - + - - - +

Correlation = -1

Range Determination

SHIFT RECEIVED SIGNAL TO THE RIGHT BY THREE CHIPS

.. − + + + − + + − + − − − + ...

− + + + − + + − + − − − +

PERFECT MATCH !

Correlation = 13 = number of chips in code

Range Determination

Shift of 3 chips

Each chip has duration τ seconds

So distance between satellite and receiver is c x 3τ

Can subsample the pulse so can measure delays to fractions of a chip duration

Inaccuracy in satellite and receiver clocks – above range estimate called “pseudorange”

Corrected by the data message in the incoming signal

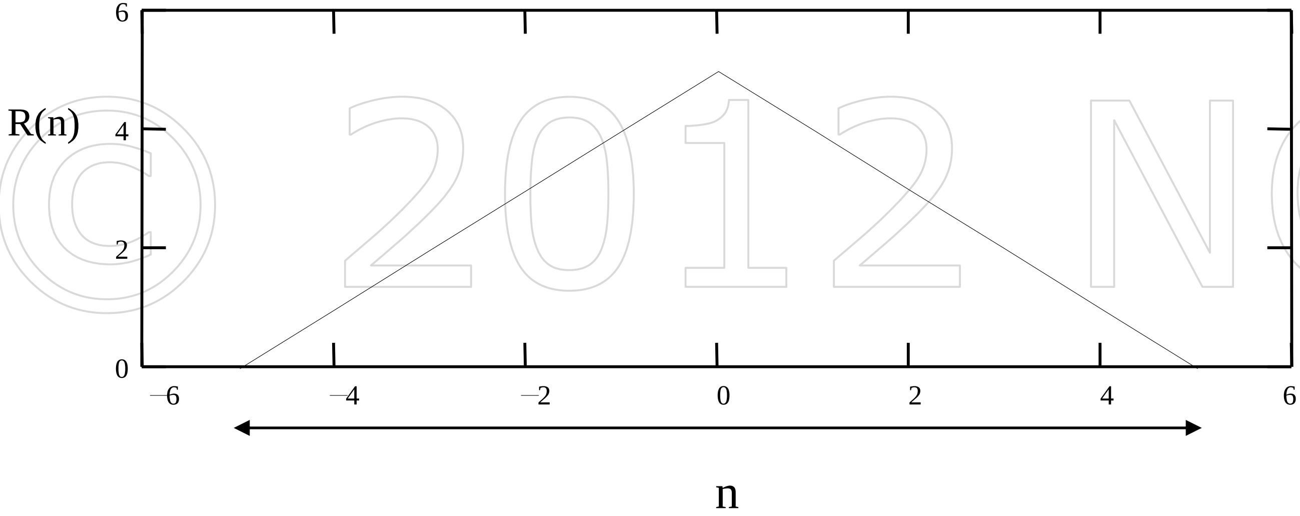
Accuracy and Resolution

Coding of pulse gives improved resolution of closely spaced objects compared to uncoded pulses

Can think of an uncoded pulse as consisting of all +'s

Accuracy and Resolution

Autocorrelation function looks like



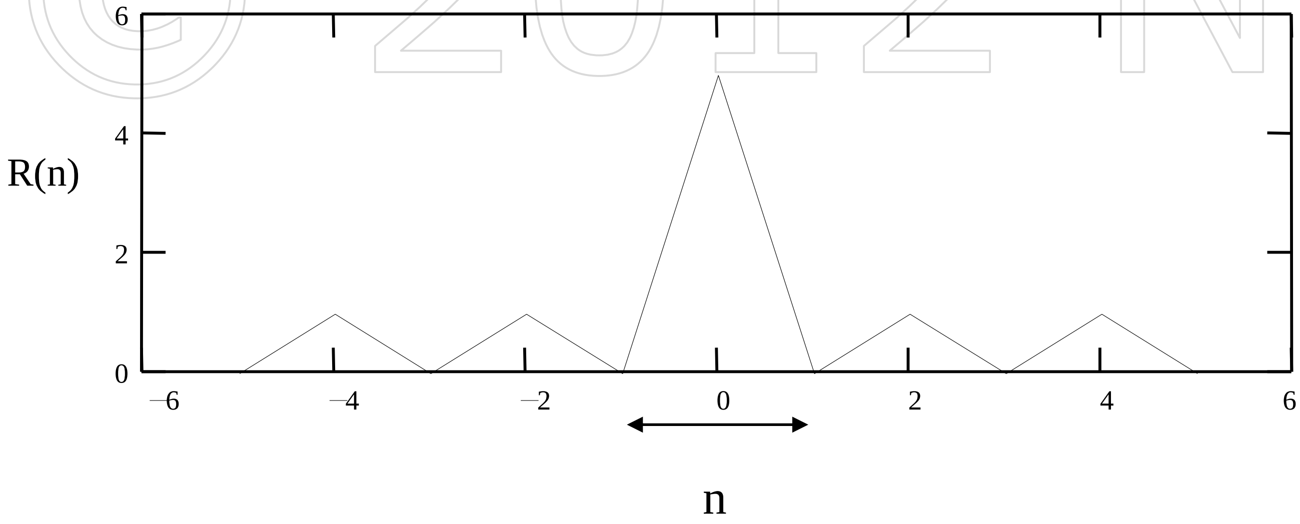
Relatively broad peak

Accuracy and Resolution

Coded pulse

+	+	+	-	+

Autocorrelation function



Narrower peak than uncoded pulse !

Accuracy and Resolution

For good accuracy of range estimate require a coded pulse with autocorrelation function with as small sidelobes as possible compared with the mainlobe amplitude.

Gold codes are appropriate, having low autocorrelation function sidelobes for given number of chips

Correlation can do this

Example:

Satellite 1 transmits

+	+	+	-	+

Satellite 2 transmits

+	-	-	+	+

Want to pick up Satellite 1's transmission at the receiver

Correlate

+	+	+	-	+

stored in the receiver with each of these coded pulses from the satellites.

Distinguishing between Different Satellite Transmissions at Receiver

Correlation can do this

Satellite 1 transmits	+	+	+	-	+
correlated with	+	+	+	-	+

in the receiver leads to the following autocorrelation function:

{1,0,1,0,5,0,1,0,1}

Maximum value = 5

Correlation can do this

Satellite 2 transmits	+	-	-	+	+
correlated with	+	+	+	-	+

in the receiver leads to the following autocorrelation function:

{1,2,1,-2,-1,2,1,-2,1}

Magnitude of maximum value = 2

Distinguishing between Different Satellite Transmissions at Receiver

Template stored in receiver for a particular satellite has maximum correlation value with transmission from that satellite

This template gives lower maximum values for the cross-correlation function when correlated with transmissions from other satellites

Hence ideal set of codes has the following properties:

As small as possible sidelobes in autocorrelation function compared to mainlobe – good range accuracy

As small as possible cross-correlation between pairs of different codes

Hence Gold codes preferred

GPS/Galileo and Digital Correlation

Advantages of digital coding in GPS/Galileo:

- Signal to Noise ratio (SNR <<1). Correlation methods can detect signals in most cases because of their robustness to noise.
- Narrowness of peak in autocorrelation function means high range accuracy.
- One can restrict the use of GPS to certain users, because the user has to have a special code in the receiver in order to correlate with the codes transmitted by the satellites.

GPS/Galileo and Digital Correlation

Because SNR <<1 for the signals transmitted by the satellites, then they cause minimal interference to signals in other applications.

Techniques can be adopted to reduce the problems with spoofing (transmission of standard codes by rogue transmissions) by encryption of the P-code.

Transmitted Signal

How do we combine ranging codes with Message?

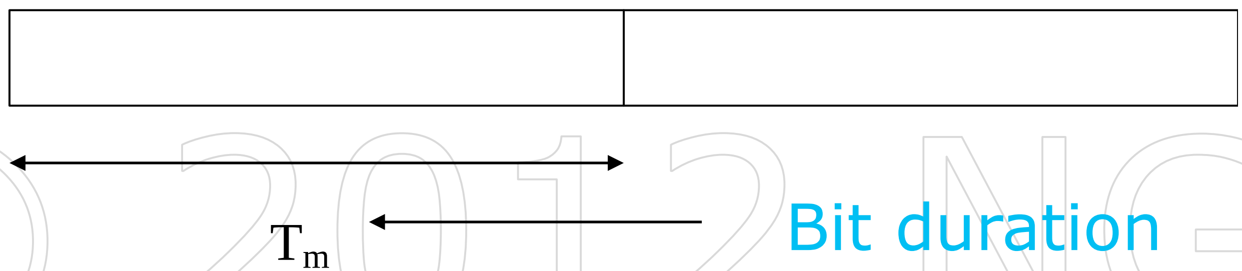
How do we transmit this information between satellite and receiver?

The technique used is Code Division Multiple Access (CDMA)

CDMA: Each satellite characterised by a particular ranging code.

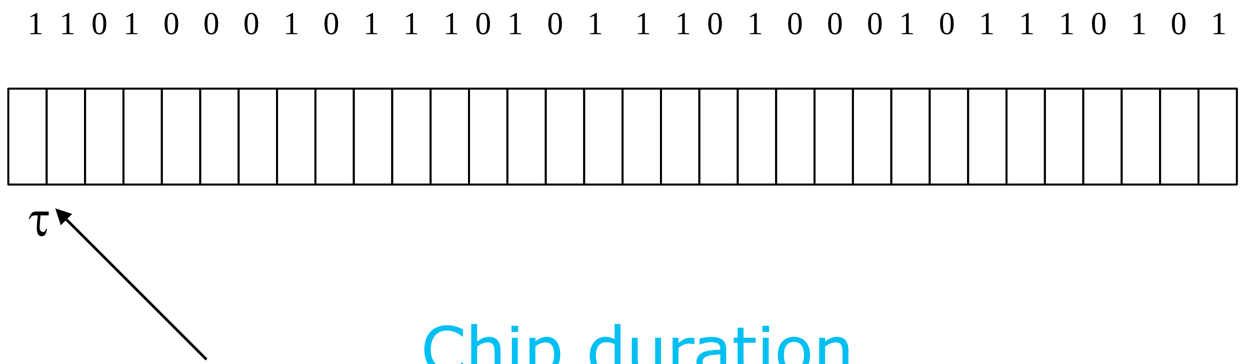
A Simple Example

Two bits of message



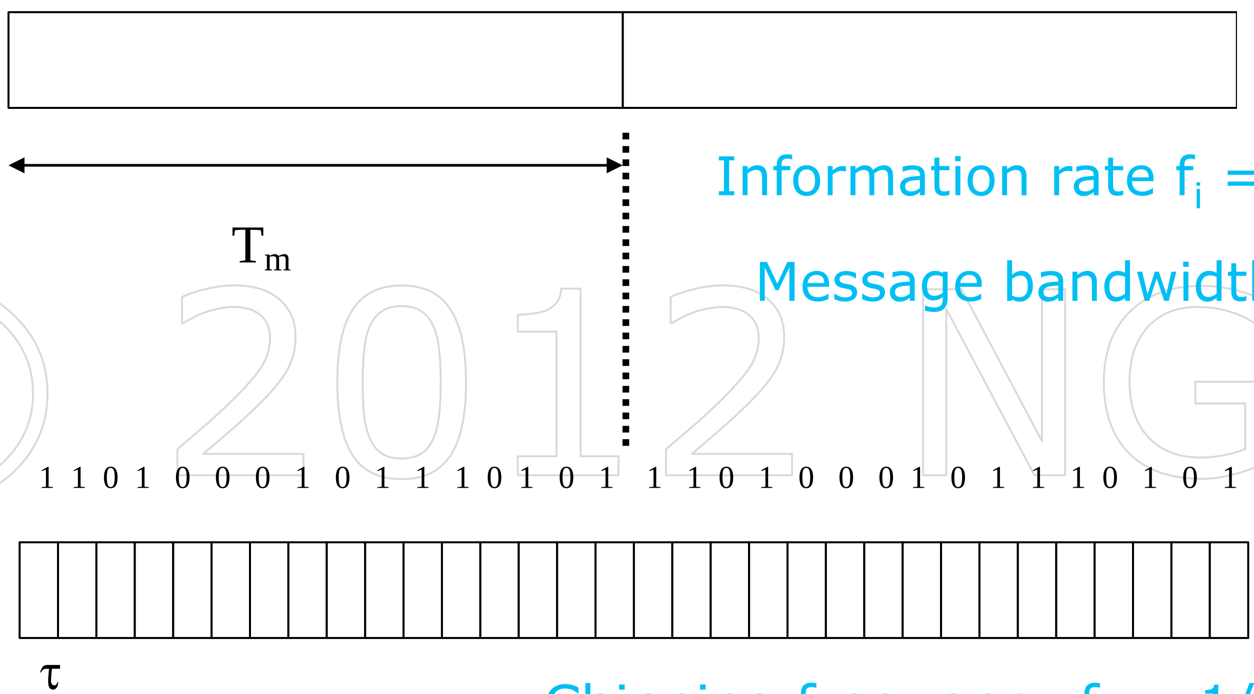
Bit duration

Two periods of ranging code, each period = 16 chips



Chip duration

A Simple Example



Information rate $f_i = 1/T_m$ Hz

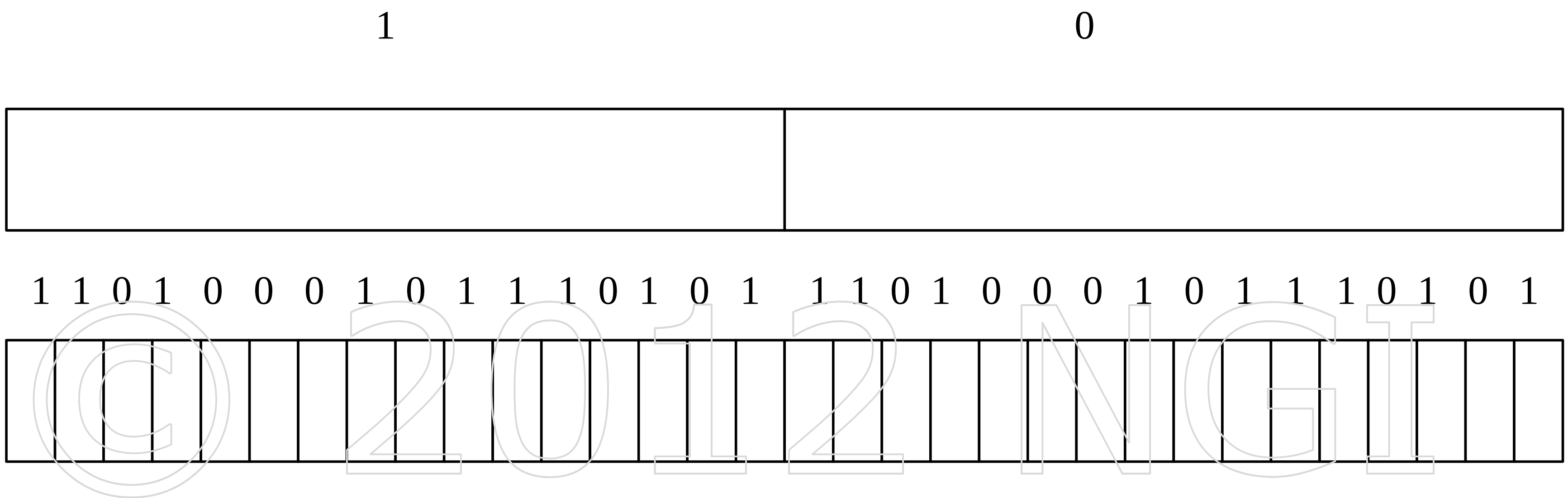
Message bandwidth $B_m \approx 2 f_i$

Chipping frequency $f_c = 1/\tau$ Hz

Pseudonoise bandwidth $B_c \approx 2f_c$

$B_c >> B_m$

Spectrum Spreading



If message bit is 1, the spread code is the "opposite" of the PN code sequence i.e. 1 → 0 and 0 → 1

If message bit is 0, the spread code is the <u>same</u> as the PN code sequence

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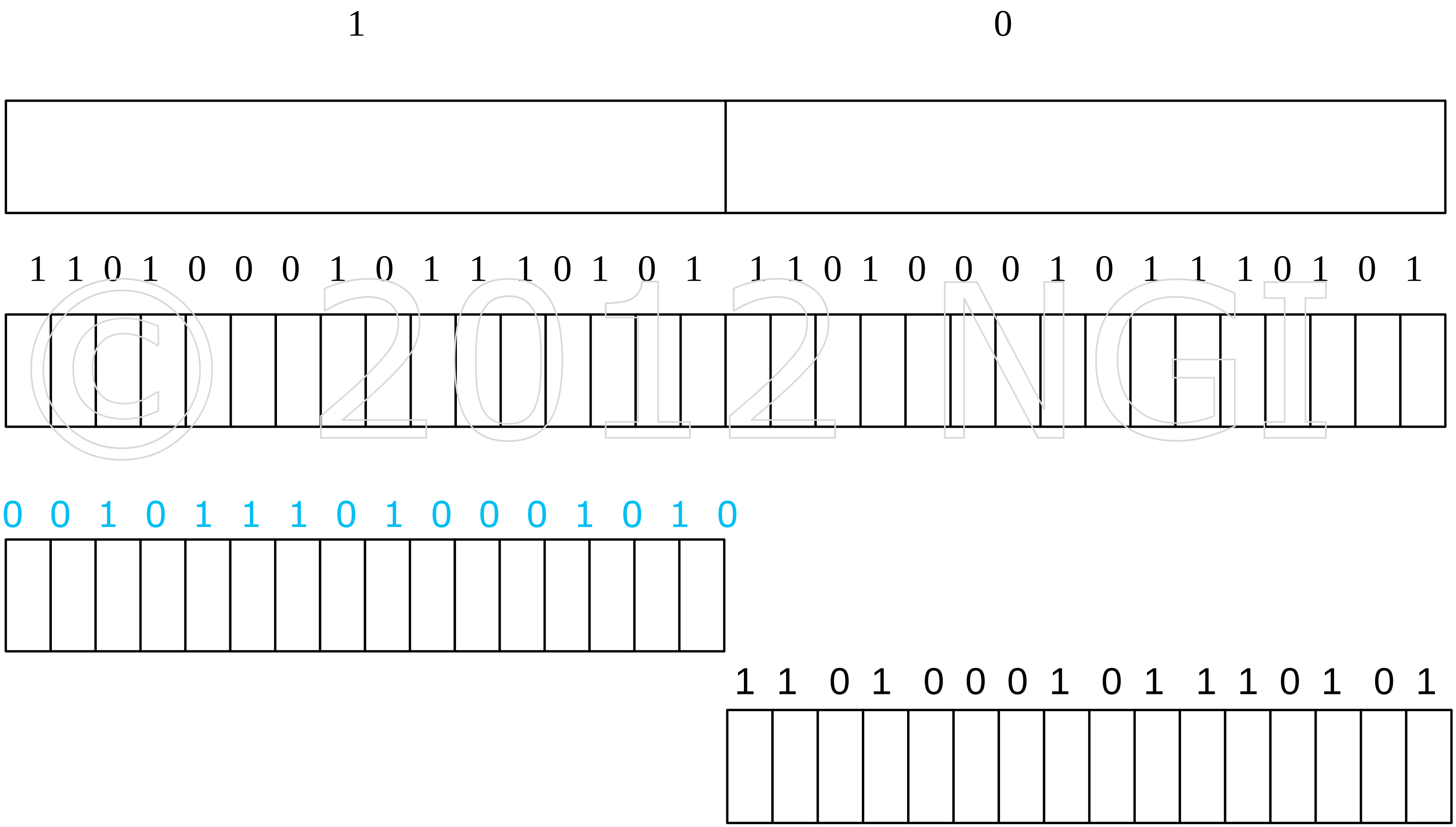
Spectrum Spreading



Spectrum Spreading

- Message "modulates" the PN code
- Spread codes contains information on ranging code and message – "modulated code"
- Modulated code has same bandwidth as PN code.
- Message has been "spectrally spread"
- Modulated code more immune to noise than original message

RF Carrier Modulation

Need to transmit Spread Sequence at higher frequencies in order to use practically sized antenna

Unmodulated Carrier

$s_c(t)=\sin(2\pi f_c t)$

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RF Carrier Modulation

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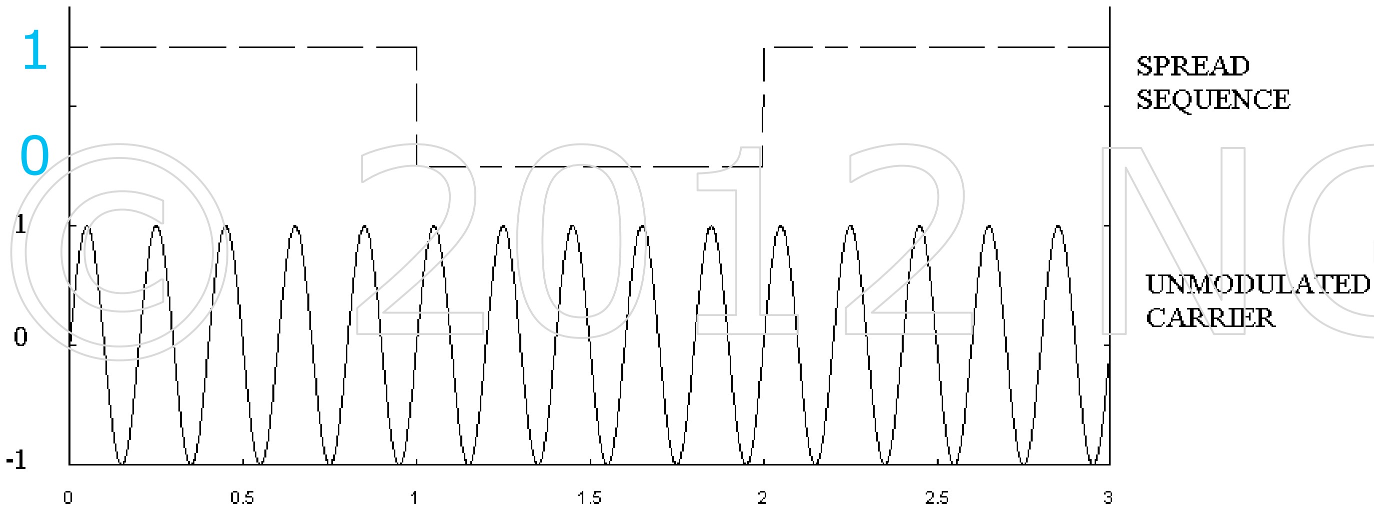

If Spread Sequence = 1, modulated carrier = – unmodulated carrier

If Spread Sequence = 0, modulated carrier = unmodulated carrier

RF Carrier Modulation

Multiplying a sine wave by -1 is equivalent to shifting it in phase by π

Hence associate phase π with a 1 in the spread sequence and a phase of 0 with a 0

Using two phases, 0 and π, to modulate the carrier

Refer to this technique as Binary Phase Shift Keying (BPSK)

GPS Signal Structure

$C_M(t)$ = Message Spread by C/A code

$P_M(t)$ = Message Spread by P code

Two signals transmitted by satellite:

$$L1(t) = a_1 C_M(t).\cos(2\pi f_1 t) + a_1 P_M(t).\sin(2\pi f_1 t)$$

$$L2(t) = a_2 P_M(t).\cos(2\pi f_2 t)$$

f_1 = 1575.42 MHz

f_2 = 1227.6 MHz

Extraction of Information at Receiver

L1 and L2 signals that are incident on the receiver

$$L1(t) = a_1 C_M(t).\cos(2\pi f_1 t) + a_1 P_M(t).\sin(2\pi f_1 t)$$

$$L2(t) = a_2 P_M(t).\cos(2\pi f_2 t)$$

Extract:

- $C_M(t)$
- $P_M(t)$ (if permitted)
- Message
- Carrier Phase

Extraction of Information at Receiver

Use correlation techniques to simultaneously estimate pseudorange and extract Message bit-by-bit

Example:

Modulated Code : 0 0 1 0 1 1 1 0 1 0 0 0 1 0 1 0 1 1 0 1 0 0 0 1 0 1 1 1 0 1 0 1

PN Sequence : 1 1 0 1 0 0 0 1 0 1 1 1 0 1 0 1

Shift PN Sequence so that correlation with Modulated Code is either +N or –N where N is the number of chips in one repetition of the PN code

In above example N = 16 and correlation = -16

Time shift τ can be used to extract pseudorange

Correlation is -16 indicates that the message bit is a '1'

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Extraction of Information at Receiver

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Update correlation as message comes in until the following alignment occurs

Modulated Code : 0 0 1 0 1 1 1 0 1 0 0 0 1 0 1 0 1 1 0 1 0 0 0 1 0 1 1 1 0 1 0 1
PN Sequence : 1 1 0 1 0 0 0 1 0 1 1 1 0 1 0 1

After a further time a peak correlation = +16 is obtained
Time shift τ can be used to update pseudorange

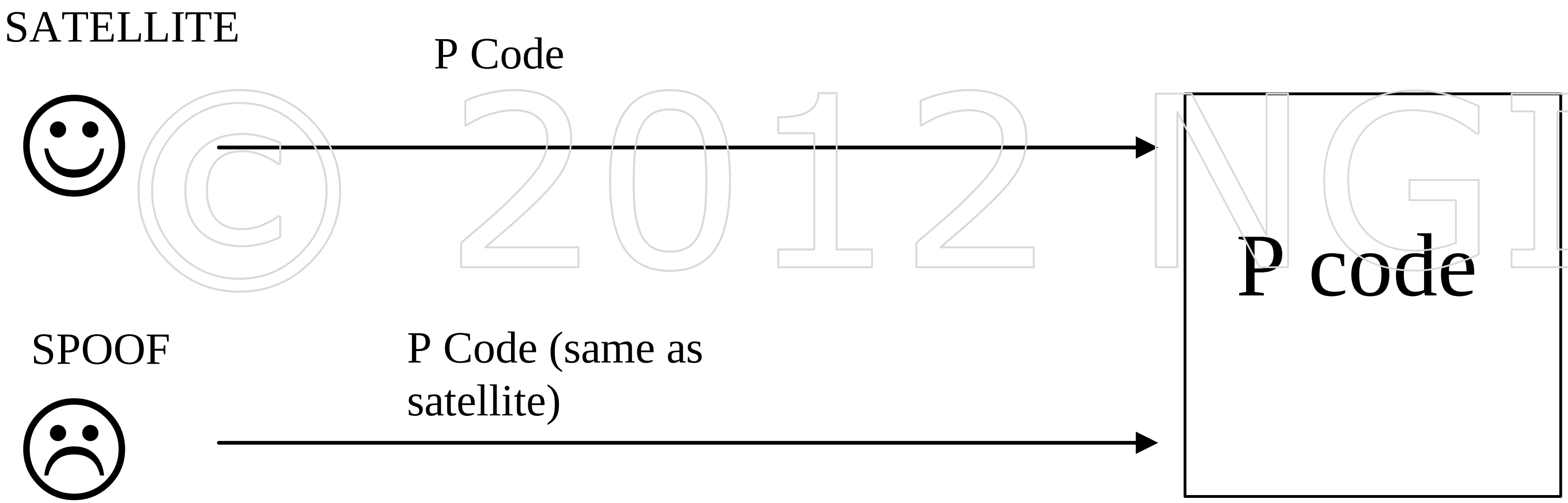
The fact that the correlation is +16 indicates that the message bit is a '0'
This process continues, each time a peak +N or –N occurs in the autocorrelation, one can simultaneously extract pseudorange and extract further message bits.

Extraction of Information at Receiver

- Extract C/A code to obtain pseudorange
- If available, extract P code to enhance pseudorange estimate
- Use information from message to obtain enhanced estimate of range
- After extraction of carrier, PN codes and message left with carrier phase – even more accurate range estimates

Anti-Spoofing

The following problem may occur:



Correlation with "Spoof" signal will lead to wrong range estimates

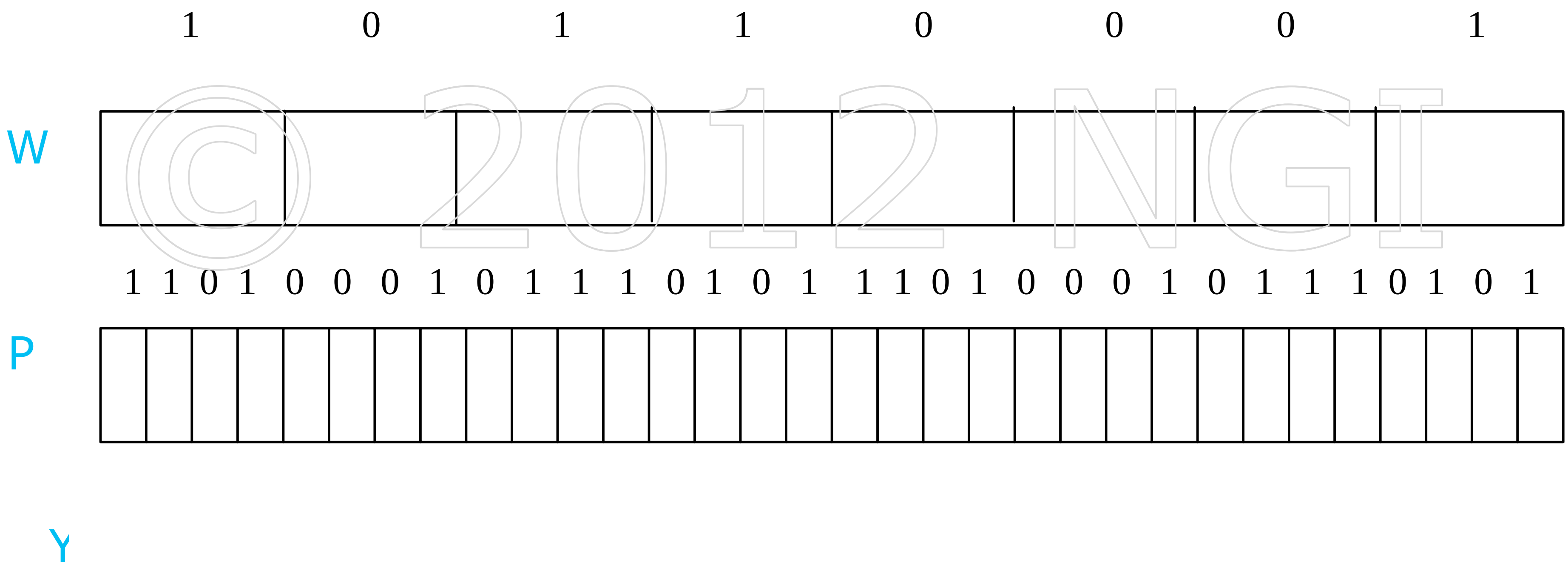
W-code

- Use W-code which is known only to the transmitter and receiver (not published!)
- W-code is non-periodic and hence unpredictable
- Clock frequency = 0.5115 MHz
- First the P-code spreads the W-code to form the Y-code

Spectrum Spreading

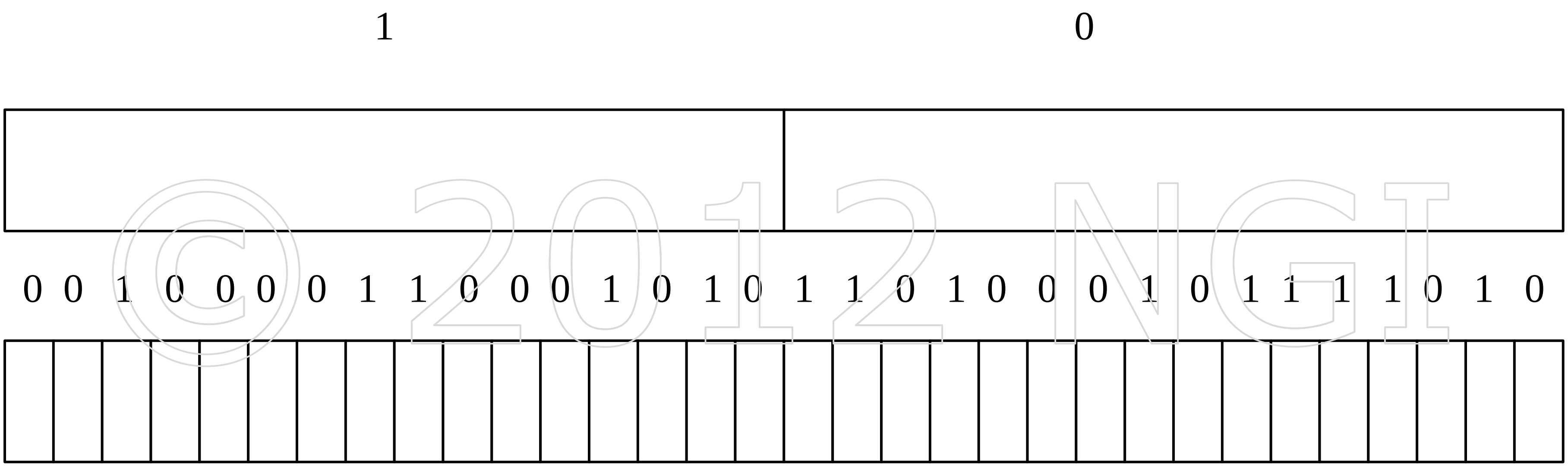
Use P-code to spread W-code

Obtain Y-code which is non-periodic and unpredictable



Spectrum Spreading

Now use Y-code to spread the message



Spread code more unpredictable than if P code had been used to spread the message

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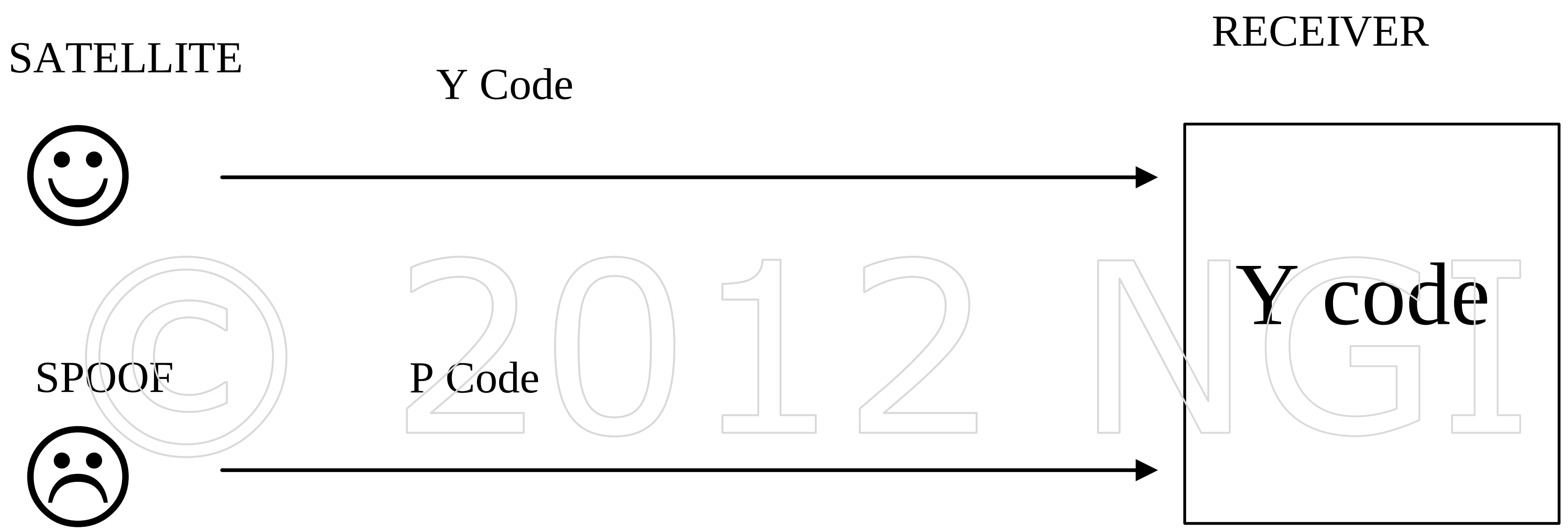
Anti-Spoofing



Y-code can be reproduced in the receiver (where W-code is known)

P-code from Spoof will not correlate with Y-code stored in receiver so will not have an effect

Lecture 2

- Digital coding and its use in satellite positioning
- Digital Correlation
- Advantages of correlation in satellite positioning
- Transmission of digital signals – modulation of high frequency carrier (BPSK)
- Extraction of positioning and other information at the receiver